

Review 15 FTC

$$I \quad \int_a^b f(x) dx = F(b) - F(a)$$

$$\int_a^b f'(t) dt = f(b) - f(a) \quad \text{net change}$$

$$II \quad \frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$$

accumulation function

$$\frac{d}{dx} [F(x) - F(a)] = F'(x) = f(x)$$

Mar 13-7:29 AM

$$Ex 1 \quad \int_1^e 1 - \frac{2}{x} dx$$

$$x - 2 \ln x \Big|_1^e = (e - 2 \ln e) - (1 - 2 \ln 1)$$

$$= e - 2 - 1 + 0 = e - 3$$

$$Ex 2 \quad \frac{d}{dx} \int_{\pi/2}^{x^2} \sin(t^2) dt = 2x \sin(x^2)^2$$

$$\frac{d}{dx} \left[F(t) \Big|_{\pi/2}^{x^2} \right] = \frac{d}{dx} [F(x^2) - F(\pi/2)]$$

$$= 2x F'(x^2) - 0$$

Mar 13-8:15 AM

$$Ex 3 \quad \text{let } F(x) = \int_1^x f(t) dt$$

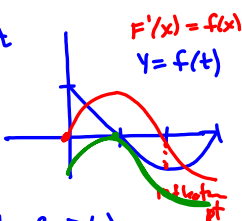
a) where does $F(x)$ have its max value? explainb) where does $F(x)$ have its min value?c) sketch a possible graph of $F(x)$ a) at $x=1$ $F'(x)$ changes from + to - at $x=1$

F accumulates pos area until $x=1$
then it gathers neg area

b) at $x=3$

F continues to accumulate neg area
until $x=3$, so F decreases until

F' is neg from $x=2$ to $x=3$
so F decreases until $x=3$



Mar 13-8:24 AM

$$Ex 4 \quad \text{Calc ok} \quad \frac{dy}{dx} = \sin(x^2) \quad y(0)=1 \quad y(\pi)=?$$

method I (doomed from the start)

$$y = \int \sin(x^2) dx = ???$$

$$\text{method} \quad y(\pi) = y(0) + \int_0^\pi \sin(x^2) dx$$

$$= 1.77265$$

Mar 13-8:46 AM

Review is FTC

I $\int_a^b f(x) dx = F(b) - F(a)$ $F(x) = \int f(x) dx$

$\int_a^b f'(x) dx = f(b) - f(a)$

↑
rate of change
of f

net change in f
on $[a, b]$

$f(a) + \int_a^b f'(x) dx = f(b)$

initial value

net change
in f

final value

Mar 13-8:55 AM

accumulation function

II $\frac{d}{dx} \int_a^x f(t) dt = f(x)$

$\frac{d}{dx} [F(x) - F(a)] = F'(x) - 0 = f(x)$

Ex 1 $\int_1^e 1 - \frac{2}{x} dx = x - 2 \ln x \Big|_1^e$

$= (e - 2 \ln e) - (1 - 2 \ln 1)$

$= e - 2 - 1 + 0 = e - 3$

Mar 13-9:51 AM

Ex 2 $\frac{d}{dx} \int_{\pi/2}^{x^2} \sin(t^2) dt$ $F' = \sin t^2$

2x · sin(x²)

$\frac{d}{dx} F(t) \Big|_{\pi/2}^{x^2} = \frac{d}{dx} [F(x^2) - F(\frac{\pi}{2})]$

$2x F'(x^2) - 0$

$f(t) = \sin t^2$

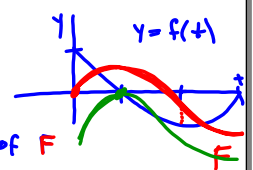
$f(x^2) = \sin(x^2)^2$

Mar 13-9:59 AM

Ex 3 Let $F(x) = \int_{-1}^x f(t) dt$

$\int_1^0 = -\int_0^1$

- a) where does $F(x)$ have a maximum? explain
- b) where does $F(x)$ have a minimum? explain
- c) sketch a possible graph of F



- a) max at $x=1$ $F'(x) = f(x)$, $f(x)$ changes from + to - at $x=1$
- b) min at $x=3$ $f(x) < 0$ from $x=1$ to $x=3$
so $F(x)$ decreases from $x=1$ to $x=3$

Mar 13-10:07 AM

Ex 4 $\frac{dy}{dx} = \sin(x^2)$ $y(0)=1$ $y(\pi)=?$

$$y(\pi) = y(0) + \int_0^{\pi} \sin(x^2) dx$$

$$= 1.77265$$

~~$$y(x) = \int \sin(x^2) dx$$~~

Mar 13-10:26 AM